TRANSPORT OF MOMENTUM AND ENERGY IN A DUCTED JET

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I. Experimental Study of a Nonisothermal Jet of Air Discharging into a Duct

A heated jet of air from an 0.898-in. standard A.S.M.E. nozzle was discharged into a 4-in. steel duct, well insulated over its entire 10-ft. length. Air from the region surrounding the nozzle was entrained into the duct. At a number of points along the duct, radial profiles of air velocity and temperature were obtained by means of a probe which combined an impact tube and a thermocouple. The temperature at each of several points along the duct wall was indicated by thermocouples imbedded in the wall.

In the experiments reported here the velocity at the jet was 585 ft./sec.; the temperature of the jet was about 220°F. and that of the entrained air was about 88°F. The total air flow rate through the duct was 0.67 lb./sec. and the heat flux was 4.9 B.t.u./sec., with the temperature of the entrained air taken as the datum.

The radial and axial profiles of velocity and temperature are compared and discussed; the temperatures of the stream near the duct wall and of the duct wall itself are given. Conservation of mass and heat was checked by graphical integration of the radial profiles.

The mixing of high-velocity gas streams is a problem of numerous facets, many of which have industrial importance. This study deals with gas mixers, jet-pumps, and ejectors—all examples of a jet discharging into a duct and entraining with it fluid from outside the duct entrance.

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APPARATUS

The equipment used to supply the air, to heat it, and to control and meter its rate of flow, has been described previously (*1*). The experimental flow section used consisted of an approach section, a flow nozzle, a duct, and a probe. Figure 1 shows the details of the duct assembly. The approach section was fitted with five screens, to eliminate abnormal flow distribution upstream of the nozzle; with a thermocouple; and with a static pressure tap. The flow nozzle was a standard A.S.M.E. long-radius nozzle, approximately 0.9 in. diam. The duct was a 10-ft. length of 4-in. O.D. steel tubing, with thermocouples soldered into the wall at intervals. The axes of the duct and the flow

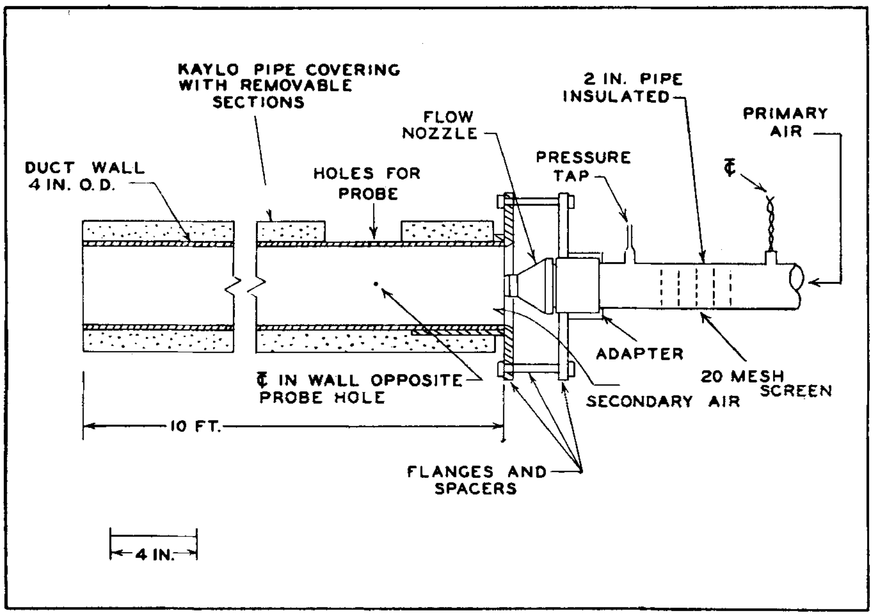

Fig. 1. Details of Duct Assembly.

Fig. 2. Temperature-impact Probe.

nozzle were aligned by means of the flanges and spacers shown in Figure 2.

The duct was insulated with a 1-in. Kaylo pipe covering to prevent heat loss. An insulating material of high structural stability was required, for it was necessary to remove sections of the insulation repeatedly in order to place the probe holder.

The probe drawn in Figure 2 was made by bending a length of 0.065-in. O.D. hypodermic tubing to form a right angle. One side of the angle was 1.0 in. long with the open end carefully machined to the proportions recommended for sharp-edged Pitot tubes (2); the other side of the angle was soldered to a length of brass tube. Beyond the brass tube the thermocouple wires were cemented to the impact tube and were wound with thread to keep them in place. The wires were joined in a short twisted and soldered junction, placed just beneath the mouth of the impact tube. The positioning device on which the temperature - velocity probe was mounted was machined to fit on the outside surface of the duct; a scale was attached to the positioning device. A pointer was attached to the probe so that when the pointer was placed at the edge of the scale on the positioning device, the axis of the impact tube was oriented parallel with the axis of the duct, and the scale indicated the radial position of the impact tube. The probe was inserted into the duct through holes drilled at intervals in the duct wall. Static pressure readings were made through the same holes by means of a tube, the end of which had been machined so as to conform closely with the inner surface of the duct.

Pressures were measured by means of a manometer system which included a mercury manometer, a water manometer, and a Meriam draft gauge. Temperature readings were obtained by means of a potentiometer and copper-constantan thermocouples.

PROCEDURE

Before any experiments were made, the apparatus was checked to assure correct alignment of duct and nozzle, and of probe and positioning scale. Such checks were also made at intervals to assure the reliability of the measurements. The holes in the duct through which the probe might be inserted were spaced fixed distances apart; the distance from these holes to the nozzle was determined to the nearest 1/16 in.

All the data reported here were obtained at an air flow rate of 2.07 std. cu. ft./sec. The flow rate was indicated by the temperature and pressure upstream of the critical orifice meter. The metered air passed through heat exchangers where it was heated to 222° to 225°F. measured upstream of the flow nozzle.

One run was made to establish each profile. Several times during each run the pressure and temperature upstream of the critical orifice, the pressure and temperature upstream of the flow nozzle, the room temperature, and the barometric pressure were all checked to discover variation from the desired mean values. At the beginning and end of each run the static pressure probe was used to measure the static pressure at the duct wall. It was assumed that the static pressure was invariant across the duct. Since the axial positions at which static pressure was measured did not coincide with the positions of the profiles, the static pressures at various points along the wall were plotted vs. distance from the nozzle, and the pressure corresponding to the position at which the profile was determined was selected from a smoothed curve through the measured values. The wall temperature, T_w, was observed several times in each run.

A traverse was performed as follows. The probe was inserted into the desired hole in the duct and raised to its highest position at the top of the duct; then its position on the scale was noted. Next it was lowered to the bottom of the duct and that position noted. The axis position was assumed to be midway between the two extremes. In the course of preliminary runs this procedure was checked against the pressure profiles obtained, and in each case the geometrical centers coincided. Because of undue disturbance of the flow pattern by the brass tube supporting the impact tube when the probe was located below the center line of the duct and because of the symmetry of the duct, the profiles were observed only from the duct axis upward to the top of the duct.

INSTRUMENTATION

Velocity was measured by means of the modified impact tube, with the effect of turbulence upon the impact tube being ignored. Since the presence of the thermocouple immediately beneath the opening of the impact tube was thought likely to disturb the behavior of the instrument, the modified impact tube was checked by calibration against an ordinary impact tube. The presence of the thermocouple was found to have no noticeable effect. Hence, the velocity was calculated from the difference between the observed impact and static heads by the use of the calculated impact tube coefficients given by Taylor, Grimmett, and Comings(*10*).

The thermocouples soldered to the duct wall were checked against each other and against the probe thermocouple at three temperatures, and at no time did they show a difference greater than 1°F.

There was some question as to the significance of the temperature indicated by the probe thermocouple. At low velocities the indicated temperature, T_{in}, should approximate the free-stream temperature; at higher velocities the temperature should be between the stagnation temperature, T_s, and the free-stream temperature, T_f. Hottel and Kalitinsky(*7*) indicate that for twisted thermocouples the recovery factor should be between 0.79 and 0.83; that is,

$$\frac{T_{in} - T_f}{T_s - T_f} = R, 0.79 < R < 0.83 \quad (1)$$

The flow nozzle used in these experiments was designed to permit isentropic flow, and so the stagnation temperature downstream of the nozzle should be equal to the gas temperature upstream of the nozzle. Because the temperature upstream of the nozzle, and hence the stagnation temperature immediately downstream, were known, it was possible to determine the recovery factor experimentally. In one case the upstream temperature was 221°F., the temperature indicated by the thermocouple was 216°F., and the gas velocity was 585 ft./sec. The relationship among stagnation temperature, free-stream temperature, and velocity was based on the assumption that the stagnation temperature is obtained when all the kinetic energy of the flowing gas is converted to heat; that is,

$$T_s = T_f + \frac{\bar{u}^2}{2\, g_c \cdot 778 \cdot C_p}$$

where 778 ft. lb. force/B.t.u. is the mechanical equivalent of heat. By taking the specific heat of air to be 0.237 B.t.u./(lb. mass)(°F.), one obtains from this equation

$$T_s = T_f + \frac{\bar{u}^2}{11{,}900}$$

where the temperatures are in °F., and the velocity is in ft./sec.

For this case, then, the calculated free-stream temperature is

$T_f = 221 - (585)^2/11{,}900$
$= 193$°F., and $R = 0.82$.

CALCULATIONS

The results were computed by means of the following equations:

$$\overline{\rho u^2} = 2 g_c C_v^2 (5.2) \left[\Delta P + P_{atm} - \bar{P} \right] \quad (2)$$

$$\bar{u}^2 \cong \frac{\overline{(\rho u^2)}}{\rho} \quad (3)$$

$$\rho = \frac{(13.5\, P_{atm} + \bar{P})\, 5.2}{53.35\, (T_f + 460)} \quad (4)$$

$$T_f = T_{in} - (0.82) \frac{\bar{u}^2}{11{,}900} \quad (5)$$

$$T_s = T_f + \frac{\bar{u}^2}{11{,}900} \quad (6)$$

$$W = 2\pi \int_0^{D/2} \rho \bar{u}\, r\, dr \quad (7)$$

$$\Delta T = T_s - T_i \quad (8)$$

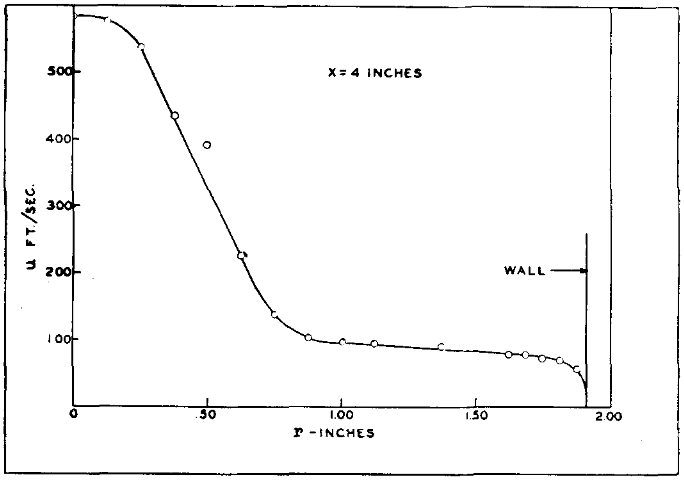

FIG. 3. TYPICAL VELOCITY PROFILE.

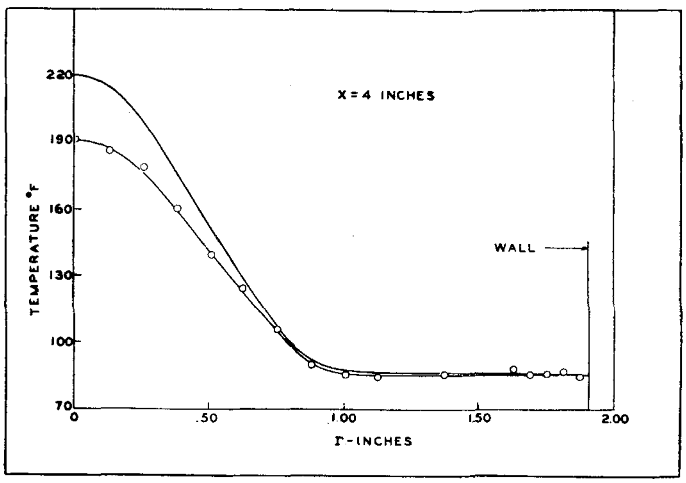

FIG. 4. TYPICAL PROFILE OF STREAM (LOWER CURVE) AND STAGNATION (UPPER CURVE) TEMPERATURE.

$$H = 2C_p \pi \int_0^{D/2} \rho \bar{u} \, \Delta T \, r \, ar \quad (9)$$

A trial and error procedure was necessary to solve for ρ, $\bar{u}$ and T_f. First C_f was assumed to be 1. Then $\overline{\rho u^2}$ was found from Equation (2). If T_f was assumed equal to T_{in} then ρ could be calculated from Equation (4). Then $\overline{u^2}$ was obtained by Equation (3). By the use of that value of $\overline{u^2}$, C_f was obtained(*10*), and T_f was found from Equation (5). Final values were obtained by iteration.

At each value of x the velocity was plotted as a function of r and a faired curve was drawn. Similar plots were made for T_f and T_s. In subsequent calculations the values of $\bar{u}$, T_f, and T_s were taken from the faired curve. Figures 5 and 6 are examples of such plots.

RESULTS

The experimental data are listed in Tables 1 through 4*. Profiles of temperature and velocity have been plotted in Figures 5 through 8. Those parts of the velocity profiles near the center of the duct resemble free-jet profiles (Figures 5 and 6). This resemblance persists down the duct until the jet spreads far enough to be affected by the presence of the duct wall. If the radial jet boundary is defined as the point at which the jet property being considered appears first to have been imparted to the entrained air stream, the jet boundary reached the wall at a point between 10 and 16 in. from the nozzle. The potential cone extended about 5 in. from the nozzle.

The temperature profiles are similar to the velocity profiles in the region near the duct axis, as seen from Figures 7 and 8. It is apparent that the jet boundaries defined in terms of stagnation temperature reach the duct wall sooner than the boundaries defined for velocity. The potential cone is shorter for the temperature case than for velocity. These phenomena are expected, in view of the experimentally observed fact that heat is transported more readily in turbulent systems than is momentum. The temperature profile across the nozzle is not nearly so flat as the velocity profile. This may be attributed to the impossibility of adequately insulating the nozzle to prevent heat flow through it. A similar phenomenon has been reported by Landis and Shapiro(*8*).

*Tables 1 through 4 (I A-D) are on file with the American Documentation Institute, Auxiliary Publications, Library of Congress, Washington 25, D. C., and may be ordered as document 4485 for $1.75 for microfilm and $2.50 for photoprints.

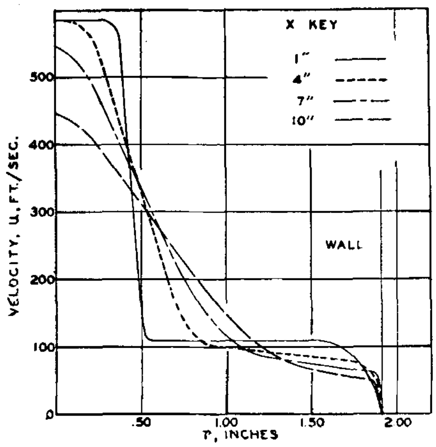

FIG. 5. VELOCITY-RADIAL PROFILE CLOSE TO NOZZLE.

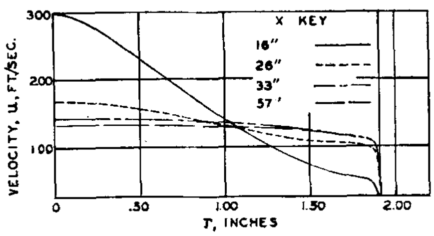

FIG. 6. VELOCITY-RADIAL PROFILES BEYOND $x = 10$ IN.

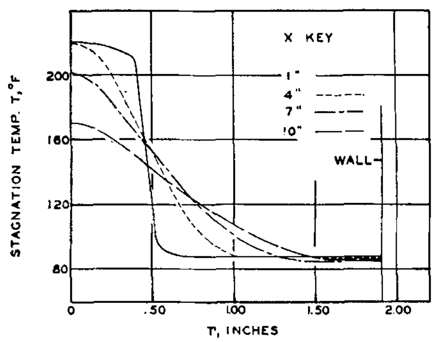

FIG. 7. TEMPERATURE-RADIAL PROFILES CLOSE TO NOZZLE.

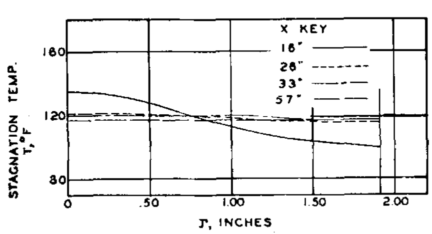

FIG. 8. TEMPERATURE-RADIAL PROFILES BEYOND $x = 10$ IN.

The axial profiles of velocity and temperature are compared in Figure 9. It is evident that the temperature begins to fall off before the velocity and that the rate of fall is greater.

In Figure 10 the stagnation temperature of air near the duct wall is compared with the duct wall temperature at various points. At 33 in. the air temperature rises smoothly to its maximum, the temperature of the fully mixed stream. The duct temperature is constant immediately downstream from the nozzle, rises to a maximum, then decreases to a minimum, and finally levels off at about 30 in.

The mass flux was determined at several values of x by plotting $\rho\bar{u}$ vs. r^2 and integrating under the curve. Figure 11 shows three such plots. Figure 12 indicates that the average mass flux was 0.67 lb./sec. and the most of the points deviate only slightly from the average. Heat flux was determined in a similar manner by plotting $\rho\bar{u}\ \Delta T$ against r^2 and integrating under the curve (Figure 13). The average heat flux was 4.9 B.t.u./sec. (Figure 14). The points scatter rather more than do those for mass flux.

DISCUSSION

The initial velocity profiles are similar to those of a free jet in a moving gas stream; however a significant difference occurs near the duct wall. The abrupt decrease in the velocity of the entrained air close to the wall cannot be attributed to wall friction at points near the duct entrance since the boundary layer at the wall should not yet have been built up; moreover, the wall friction should increase down the duct. Since the velocity gradient for the profile farthest down the duct is less sharp than for all profiles at distances less than 20 in., friction cannot explain the sharp velocity gradient near the duct wall.

For this phenomenon, previously discussed by Alexander et al.(*1*), several possible explanations can be presented. It may be due to choking, which occurs when the jet cannot discharge from the duct all the fluid induced by it. It may be due to a combination of static and impact tube effects in a region of entrainment and eddies. It may also be a consequence of the relatively sharp duct entrance past which the entrained air flowed. In experiments subsequent to those described here, Danielson(*4*), duplicating this study, used an apparatus differing from the one employed here only in that the entry section was faired, as in a subsonic induction wind tunnel. The abrupt velocity decrease was eliminated.

The temperature profiles have the expected form (Figures 7 and 8). They support the contention that heat is more rapidly transported in turbulent systems than is momentum. This has been observed experimentally by several independent investigators(*5*, *6*). The vorticity-transport theory of G. I. Taylor(*9*) was designed to present a phenomenological basis for these observations. This anomalous behavior can be allowed for readily in the mathematical treatment of free jets(*3*) by changing the value of an empirical constant in the equations for momentum and heat transport. Inasmuch as no fundamental mechanism of turbulence has yet been brought forward, the unequal rates of heat and momentum transport cannot be explained adequately.

An important difference exists between the velocity and temperature profiles. The velocity profiles indicate unusual gradients near the duct wall. However, no temperature gradients were observed at the duct wall at any cross section. The duct was insulated, and, since there was no heat flow through the wall, there should have been no temperature gradient at the wall. No reverse flow occurred; such a flow would have been from a region of high temperature to one of low temperature. Instead, the temperatures near the wall appeared to fall off slightly immediately downstream of the nozzle, before they rose because of mixing with the heated jet. This decrease in temperature might be explained as follows. The temperatures plotted on Figures 7 and 8 are stagnation temperatures, which include the effect of kinetic energy carried by the mean component of the axial stream velocity. No account has been taken of the energy carried by the turbulent velocity components, which are unquestionably large in the region being considered because of the sharp-cornered entry. If the energy involved in these fluctuating velocities could be taken into account, it is likely that the stagnation temperatures near the wall would be approximately constant until the jet boundaries reach the wall, where they should begin to rise. The only effect to be expected from the tendency for back-flow in the absence of actual reverse flow should be a change in the rate of rise of temperature. Since no data are available for the case without tendency for backflow, it is not possible to detect this tendency by comparison.

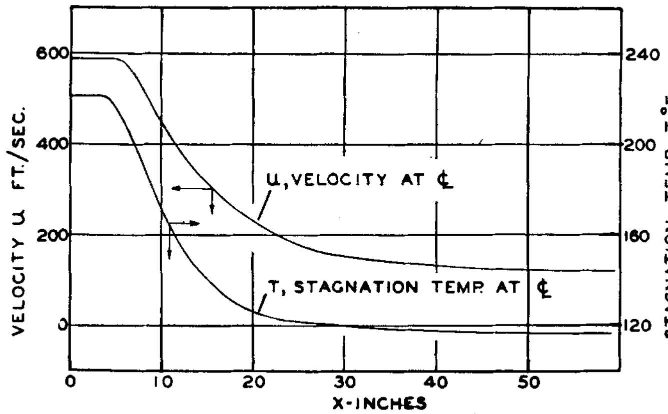

Fig. 9. Axial Profiles—Velocity and Temperature.

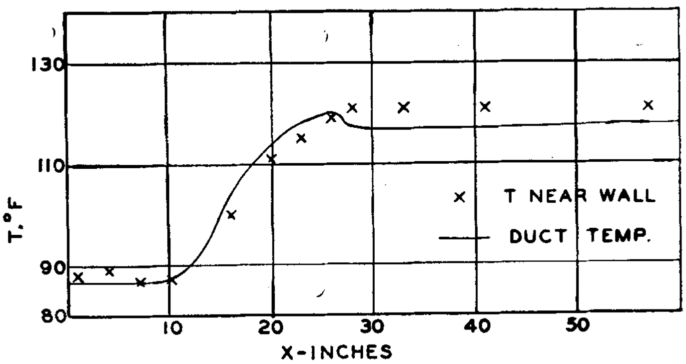

Fig. 10. Temperatures of Duct and Air Near Duct.

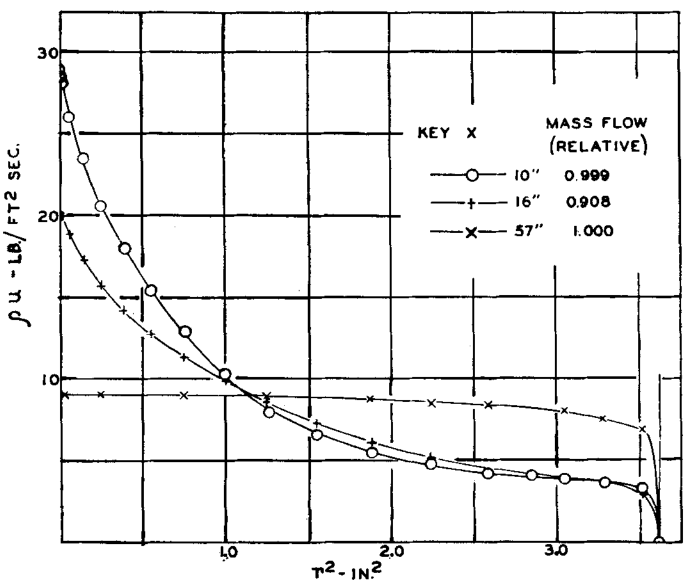

Fig. 11. Typical Mass-flux Plots.

The unusual pattern of wall temperatures is puzzling. As shown in Figure 15, this pattern was checked at various air rates. In all cases the temperature reached a maximum at 26 in. At 28 in. the temperature either leveled off or passed through a minimum in the case of higher jet velocities. This unusual behavior was not caused by erroneous thermocouple readings: the thermocouples were checked when hot air was passed through the duct, the duct was plugged, and the indicated temperatures were observed as a function of time. The temperatures were high at the downstream end of the duct and lower at the upstream end because of heat loss from the duct through the flange at the upstream end to the ambient air. But the progression of temperatures was a smooth one, indicating no erratic thermocouples. Nor was the behavior attributable to heat conduction from the duct to its supports. There were no supports in the region from 10 to 36 in. downstream of the nozzle.

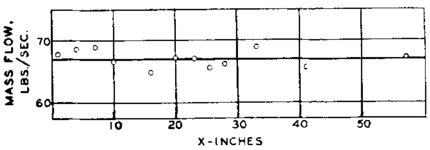

FIG. 12. MASS-FLUX VARIATION WITH x.

The following speculative mechanism may explain the wall temperatures. The wall was alternately heated by hot eddies and cooled by cold ones, and the resulting wall temperature depended more upon the maximum temperature, that of the hottest eddies, than upon the mean air temperature. This mechanism existed in the region immediately downstream from the jet, as far as 26 in. downstream. At another region of the duct the gas was mixed sufficiently so that its temperature was relatively constant, subject to minor fluctuations. In this region heat transfer to the wall from the air stream was a function of the mean temperature, and the wall temperature approached the temperature of the mixed gases. Heating by distinct eddies occurred near the nozzle; heating by the mean fluid stream occurred after the jet boundaries had spread as far as the wall. The transition between the two modes of transfer was sharp; hence the wall temperature fell sharply when the heat transfer changed from dependence upon hot, turbulent eddies to dependence on a moderately warm, low-turbulent stream.

If this explanation is correct, one would expect to find that at high velocities under conditions of greater turbulence, the effect of hot eddies would be enhanced since the heat transfer rate is increased by high turbulence. Further, the transition between the two modes of heat transfer would be more pronounced. Figure 15 indicates this to be true.

The stagnation temperature of the air close to the duct wall also presents an unusual problem. Since the wall must be heated by the air, the second law of thermodynamics would preclude the wall temperature from being higher than that of the air, as is apparently the case between 10 in. and 28 in. as indicated in Figure 10. There are several possible explanations for this phenomenon. First, because of the high turbulence present in the region close to the wall in the vicinity mentioned, the stagnation temperatures computed might not be correct, since under conditions of high turbulence the turbulent velocity components might bear a substantial portion of the total kinetic energy; the stagnation temperatures might therefore be appreciably higher than those calculated from mean velocity measurements. Second the effect of alternate hot and cold eddies on a thermocouple is not necessarily the same as the effect of those eddies on a solid wall. In either case, the calculated values would be in error.

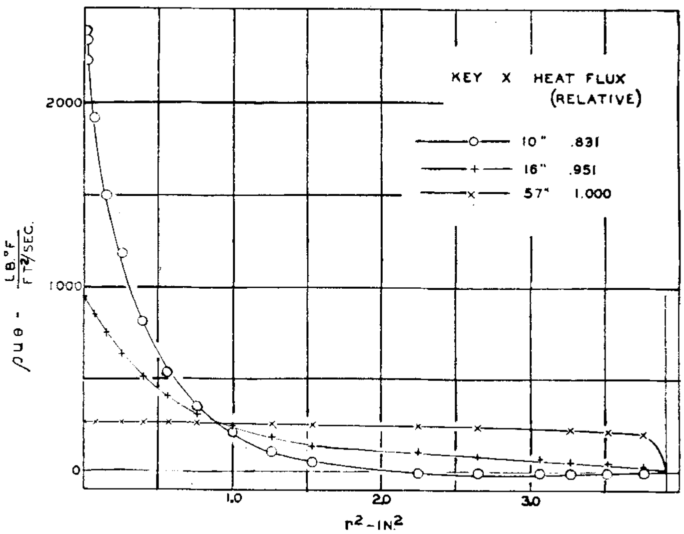

FIG. 13. TYPICAL HEAT-FLUX PLOTS.

At positions beyond $x = 28$ in. the stagnation temperature near the wall is constant at a value higher than the duct wall temperature; the stream temperature, however, is equal to that of the duct wall, as expected for an insulated wall.

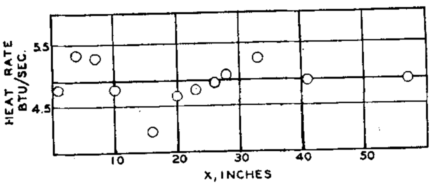

FIG. 14. HEAT-FLUX VARIATION WITH x.

The mass balance along the duct is satisfactory, Figure 12. If the value at 16 in. is disregarded, the mean deviation from the average mass flux is 1.4%, and the maximum deviation is 2.4%. the heat balance, since it must reflect the errors in temperature measurement as well as those in velocity measurement, is less precise, Figure 14. The mean deviation from the average heat flux is 4.1%; the maximum deviation is 8.6%; again, the profile at 16 in. has been disregarded.

SUMMARY

Profiles of temperature and velocity were measured for the case of a jet of air at 585 ft./sec. and 222°F. discharging into an insulated duct and entraining air at 88°F. The temperature of the wall at several points along its length was also measured. From these measurements heat and mass fluxes

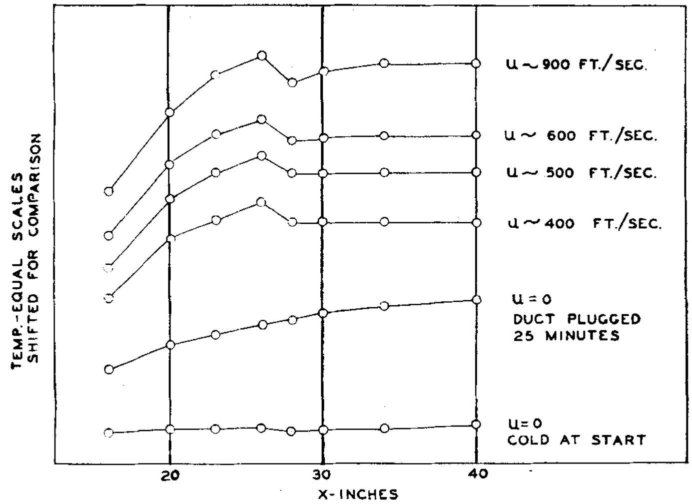

FIG. 15. VARIATION OF WALL TEMPERATURES WITH AIR RATE.

were computed at several points along the duct. The deviation of the measured values of mass flux from the mean value was small; the corresponding deviation in the case of heat flux was appreciably larger.

ACKNOWLEDGMENT

This work was performed as a part of a research program in the Engineering Experiment Station of the University of Illinois on the mixing of high-velocity gas streams. The main investigation was begun in 1946 under Contract N6-ori-71 Task XI with the Office of Naval Research. It was supported during the period from October 1, 1950, to December 31, 1951 under Contract DA-18-064-CML-445 with the Army Chemical Corps.

NOTATION

C_f = impact tube coefficient
C_p = specific heat at constant pressure
c = momentum-spreading coefficient, empirical constant in Equation (19)
g_c = 32.2 ft. lb. (mass)/lb. (force) sec.^{2}
H = energy flux at any section. B.t.u./sec.
P = static pressure in duct
P_{atm} = static pressure in atmosphere
ΔP = pressure indicated by manometer, one end connected to a total head impact tube in a duct and the other open to the atmosphere
R = thermocouple recovery factor, Equation (1)
r = radial coordinate, distance from axis of duct
T = temperature
ΔT = temperature difference, Equation (8)
u = x-directed component of velocity
W = mass flux at any section, lb./sec.
x = axial coordinate, distance along axis of duct from nozzle
ρ = density of fluid

Subscripts

atm = atmosphere
f = free stream or static condition
i = induced stream conditions in plane of nozzle
in = indicated by thermocouple
n = upstream from flow nozzle
or = upstream at orifice
s = stagnation
w = condition in duct wall

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II. Theoretical Study of Turbulent Transport of Momentum, Energy, and Matter in Ducted Coaxial Streams

This paper concerns the kinetics of the processes that take place when a high-velocity jet of fluid mixes turbulently with a low-velocity, induced stream of the same fluid in a duct of uniform diameter. Semi-two-dimensional solutions of the equations of transport involving two empirical coefficients were obtained by application of Reichardt's hypothesis and three assumptions: (*a*) a negligible fraction of the flow entity (energy, mass, or momentum) is lost at the wall and the presence of the boundary layer may be ignored, (*b*) the static pressure is uniform over a section of the duct, and (*c*) the turbulence pattern is similar to